Fuzzy Set Modeling of Insurgent Networks

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ABSTRACT

A modeling framework is presented to analyze the effects of culture on the dynamics of small to medium sized social networks. The hybrid framework utilizes fuzzy set theoretic concepts in a Monte Carlo simulation. A network is defined where the nodes are members of the network and the weight of the edges represent the subjective strength of the relationships between agents. These weights are determined by applying a fuzzy set metric to Axelrod's canonical model of culture. The network's influence on agents is determined by the relative strengths and graph theoretic distance between agents. Using simulation, interesting results were obtained for both simple abstract networks and larger social networks modeled on Al Qaeda. In conclusion, this framework provides a mechanism to model social network dynamics where both cultural traits and network topology are of principal interest.

INTRODUCTION

One of the central insights of modern sociology and applied mathematics has been the notion that influence moves through informal networks rather than through hierarchical structures. Numerous simulation studies and modeling approaches have taken this insight and applied it to a host of organizations. A key shortcoming of these models is that the central determinant of group dynamics - such as how resistant the network is to the removal of individual nodes - is the network's structure i.e., its topology. Applying these models to policy-analysis implies accepting the assumption that a social network in Kosovo will have similar dynamics to one in central Iraq. This is a very strong assumption.

Essentially, existing models that are primarily driven by topology are not well equipped to explaining variance in group dynamics. Because many networks of interest, such as insurgents, occur in multi-ethnic societies and involve coordination between different cultural groups, understanding the role that culture and identity play in a network's resiliency is important. While it is clear that cultural solidarity plays a major role in sustaining these movements, it is not clear how to incorporate it into a modeling framework. Our review of the literature shows that while analysts agree on the importance of culture, a framework for modeling its impact has not been developed. Drawing on the diverse social-science literature, we present an agent-based method for modeling the effects of culture on network resiliency. We then apply this method to insurgent movements within a multi-ethnic society.

In our model, each agent is assigned an identity in an n-dimensional cultural space. This identity is picked from a distribution and represents the various observable dimensions of identity, things like skin tone, hair color, language, and the like. Each agent is also assigned a fuzzy set around this point which defines that agent's perception of membership in its culture. We then weight the influence of the agents based on cultural similarity defined through their fuzzy identity sets.

Because we define culture using fuzzy sets, the dimensions of culture can be specified according to expert opinion; there is no requirement that they all be on the same scale. We have the freedom to use continuous, ordinal, or even categorical differentiae. Additionally, the framework allows one to consider the impact of different distributions of culture and different rules for how culture maps into political or social influence. Our main findings are that when influence is mediated by cultural similarity, then culture is a more important determinant of resilience than network topology.

The remainder of the paper proceeds as follows. First, we review the existing models of social network influences on political mobilization and examine attempts to understand the role of culture in that process. Second, we present our model. Third, we provide some numerical examples and tentative results for both simple abstract networks and a real world network of insurgents. Finally, future work and limitations of the formalism are discussed.

Motivation

Mathematical models of insurgencies fall into two main types: (1) Stochastic models which look at aggregate group behavior (Lohamnn 1994; Bearman & Kim 1997; Chwe 1999; Kuperman & Zanette

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2002; and Newman 2003); (2) Micro-level models using a rational-actor approach to explain individual participation (Muller & Opp 1986 and Gates 2002).

The first approach has had some success replicating the explosive population dynamics of revolts and mass political movements. Lohmann (1994) uses the level of participation as an informative signal about the nature of the regime. As participation increases, citizens become more certain the regime is bad, so their motivation to act increases. Chwe (1999) uses a simpler threshold model where people know the propensity to revolt of those near them and revolt if they anticipate enough others will do so. He generates interesting dynamic by varying the distribution of propensities to revolt. Bearman & Kim (1997) focus on the ability of people in the network to influence others based on their relative centrality in the network. Other modeling approaches based on condensed matter physics are good for describing the global dynamics of large networks, but the models of individuals in the network are overly simplistic. (Kuperman & Zanette 2002; Newman 2003; and Vazquez, Krapivsky, and Redner 2003). None of these approaches have proven successful at explaining long-running insurgencies and offer little to guide analysts interested in predicting local patterns of membership.

The second approach suffers from slightly different problems. Muller and Opp (1986) laid out a fairly sparse model of the choice to join in rebellious collective action in which individuals can recognize the collective sub optimality of non-participation. Gates (2002) develops a model of recruitment to insurgent organizations in which government and the insurgents compete to recruit rational citizens. Both the government's and the insurgent's ability to place incentive-compatible restraints on citizens decreases relative to the distance from the geographic center of their military power. Gates notes that ideally his model would include some notion of ideological or cultural distance but is not able to quantify that idea.

More anthropologically focused studies of rebellion have pointed out that religion and local culture play a major part in motivating participation in civil wars (Kalyvas 2001). Other studies have shown that social movements look for participants in a rational fashion, starting with those who are closely connected (Brady, Schlotzman and Verba 1999).

Taken together, this diverse literature suggests that in order to understand local variation in the propensity of people to participate in insurgency requires an ability to deal analytically with the notion of social and cultural distance. For our methodology, the concept of degree in network theory provides the means for dealing with social distance. Fuzzy sets provide the means to deal with the concept of cultural or ideological distance. Axelrod's (1997) canonical computational model of culture provides the basic framework.

Axelrod begins with a simple setting, a set number of agents interacting on a square grid. The agents do not move and interact only with their neighbors immediate four neighbors (north, south, east, and west). Each agent possesses a culture made up of q features. Each feature can take one of m traits. So one feature might represent religion and the traits might represent the possible religious denominations such as Catholicism, Islam, Judaism, and the like. The similarity of two sites is simply the percentage of traits they share. In each period a site is activated at random as is one of its neighbors. They interact with probability equal to their similarity. If they interact then one of the features where the neighbor differs from the active site is selected at random and its trait switched to match the trait of the active site. From this simple setting, Axelrod is able to capture a variety of interesting dynamics including the emergence of stable regions of independent culture.

Boudourides (2003) makes the first attempts to deal formally with the influence of culture on the evolution of social networks. He first shows that within most social networks, stable independent cultures cannot survive as they do on a grid. To explain the existence of different cultures, he develops the notion of heterophilic and homophilic agents. Homophilic agents are like Axelrod's in that they seek to become more similar when they interact. Heterophilic agents create differences when they interact, changing their trait one of the features which they share with the active site.

While these models replicate the cultural diversity observed in the real world, they do not incorporate insights from psychological and ethnographic work that shows people tend to think about cultural identity and difference in terms of "fuzzy" overlapping categories. (Rosch 1975; Rosch & Muller 1978; and Mervis and Rosch 1981). Applying fuzzy set theoretic approaches to Axelrod's model allows us to incorporate this notion. Our approach should also be particularly appealing to analysts who may have access to detailed data about the demographic characteristics and affiliation tendencies of a population, but are unlikely to have the detailed network connectivity data required by traditional network analysis approaches.

Drawing on elements from the stochastic models of revolt, we propose a simple network model of insurgency participation in which members of the population participate if the social pressure to do so passes a certain threshold. This pressure is a function of both the number of others participating in the network and their influence over the individual in question. Drawing on insights from the micro-level models, we suggest this influence is mediated by both the social and cultural distance of the other participants.

Since our interest is in assessing the influence of government action on insurgencies, we begin with an established insurgent network and study the effects of various government strategies on the network given:

1. Different efficacies of government action.
2. Different levels of homogeneity of culture.
3. Different underlying networks.

The Model

Our model uses fuzzy sets to add a more subtle notion of the influence of culture than is present in Axelrod's (1997) model of culture or in Boudourides' (2003) extension of that work. A fuzzy set has the following elements: a language variable, the universe of discourse and a membership function. In our model, the language variable describes the cultural identity, Axelrod's feature. The universe of discourse describes the possible traits of the given cultural identity and the membership function maps the possible cultural traits to a real number on the unit interval [0,1]. An extensive literature on fuzzy sets exists and their application exists, for a more robust discussion on fuzzy sets the interested reader is directed to Yen (1999). The possible cultural traits that an actor can possess, or in fuzzy set theoretic terms the universe of discourse, can be represented by a set of positive integers $\{0,1,\ldots, m\}^a$. The membership function details to what degree a given point in the universe of discourse maps to the language variable.

For example, if the language variable for cultural identity is beard length, the universe of discourse are integers representing possible beard lengths, then for a given monotonically increasing membership function, an individual with a beard length of 1 has a lower membership in the set than someone with a beard length of 6. Figure (1) is a graphical representation of this concept, with membership on the y-axis and cultural location on the x-axis. Thus, using the fuzzy sets we can mathematically represent the impact of different cultural traits for any given cultural identity. It is important to emphasize that the language variable, membership function, and the universe of discourse and their properties are specified by the analyst. In the following sections we describe some different types of membership functions.

In more formal terms, for each fuzzy set $I(\lambda,\chi_{i,\ j},x)$, the language variable λ is in terms of membership, $\chi_{i,\ j}^{x}$ is the membership function for agent i in the culture of agent j along feature x. Here $\chi_{i,\ j}:\{\mu_i^x,\ \mu_j^x\} \rightarrow [0,1]$, where i, j index individuals, and the universe of discourse x is one feature in the q dimensional culture space. The values of $\{\mu_i,\ \mu_j\}$ represent the vector of observable characteristics of the agents in question, their positions in the cultural space. Note that one could also allow for exogenously given fuzzy sets centered at some "ideal" version of a type. For example, the "ideal" Serb might be at point (2,3,4), while the ideal Croat might be at (4,5,9) and we could then ask how much people are members of these ideal types.

For a given distance metric d a trapezoidal membership function for agent i in agent j's culture on feature k, $\chi_{i,j}^k$, can be written as follows:

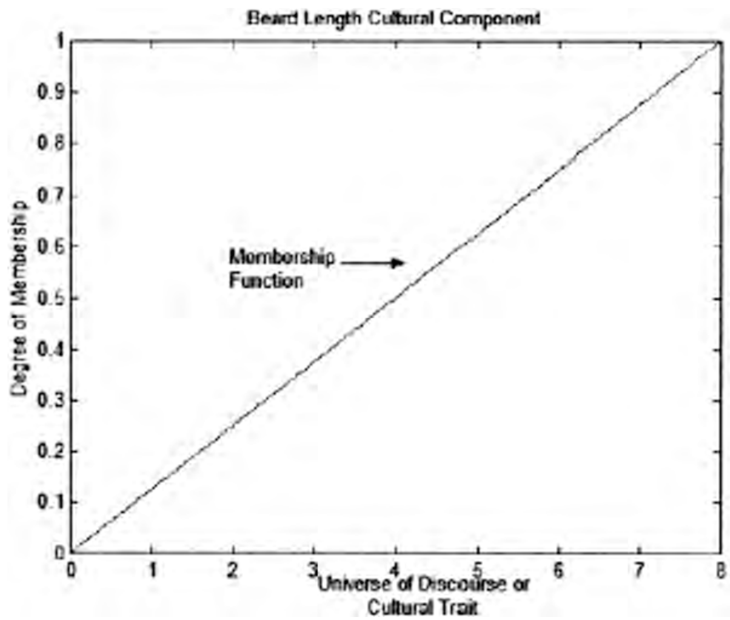

Figure 1.

$$\chi_{i,j}^{k} = \begin{cases} 0 & \text{if } d^k(i,j) > 3 \\ 1 - \frac{1}{3}d^k(i,j) & \text{otherwise} \end{cases}$$

Alternatively, different notions of how identity changes with distance from the ideal point can be modeled. For example, a the notion that you are very sure someone is part of a certain group until they reach a cut point where your confidence in their membership falls dramatically could be modeled as follows:

$$\chi_{i,j}^{k} = \begin{cases} 0 & \text{if } d^k(i,j) > 3 \\ \sqrt{1 - \frac{1}{3}d^k(i,j)} & \text{otherwise} \end{cases}$$

Or the notion that your confidence is fairly strong until $d(i, j) = 2$, but then drops rapidly for a time then tails off - think of an exponential family probability density function could be modeled using a Gaussian membership function as follows:

$$\chi_{i,j}^{k} = \begin{cases} 0 \quad \text{if } d^k(i,j) > 6 \\ \exp\left(\frac{-(\mu_i^k - \mu_j^k)^2}{(2\sigma_k^2)}\right) \text{otherwise} \end{cases}$$

Figure (2) compares the trapezoidal and Gaussian membership functions given above along one dimension with $\sigma = 1$. The dimension could represent an observable trait such as beard length. So if individual i with a beard length of 4 encounters individual j with a beard length of 2, then j's degree of membership in i's culture is 0.5 if i uses a trapezoidal membership function and is approximately 0.1 if i uses a Gaussian membership function.

The obvious alternative to our approach is to normalize d^k and combine them to define cultural influence. Such an approach is appealingly simple, but misses the fundamental utility of fuzzy sets, which permit us to model the way people actually process identity—via natural categories (Mervis & Rosch 1981). These membership functions allow some overlap between groups, thus capturing the intuitive notion that people with ambiguous identities may be considered members of more than one group. Before discussing how $\chi_{i,j}^{k}$ affects influence, a few more preliminaries are needed.

Defining the membership functions along only one dimension at a time provides important analytical flexibility. Using multi-dimensional membership functions requires making the tacit assumption that membership is evaluated in the same fashion along each feature.

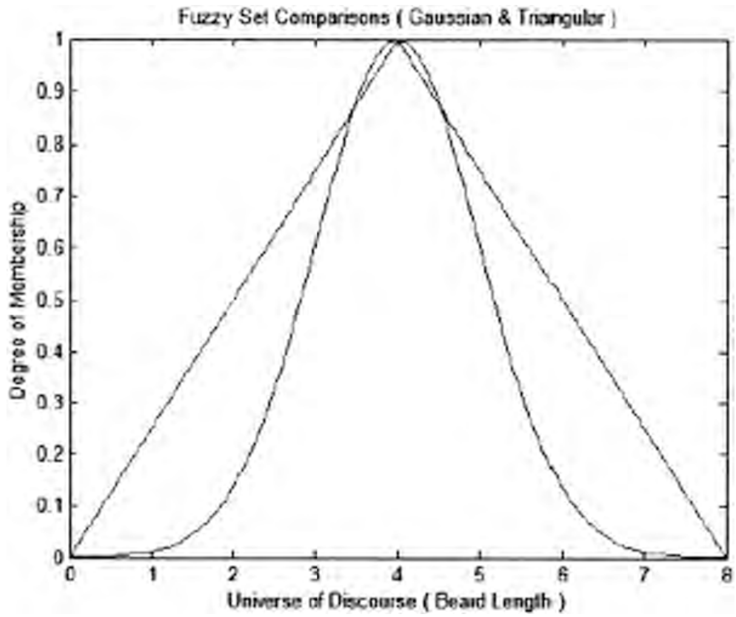

Figure 2.

Instead, we combine membership functions between features, allowing us to take advantage of the fact that cultural affinity may be evaluated differently on each dimension of culture. Not only can different membership functions be defined for each feature, but this approach allows the analyst to specify exactly how cultural traits interact. In some circumstances, cultural affinity may be evaluated as the average membership across traits. In other circumstances, the maximum membership on any trait may be used. Defining the membership functions along each dimension allows the analyst to take these possibilities into account.

The social network is an undirected graph where the connectivity matrix E is made up of elements $e_{i,\ j}$, where $e_{i,\ j} = 0 \Rightarrow$ no connection and $e_{i,\ j} = 1 \Rightarrow$ connection. Define the strength matrix S such that each element $s_{i,\ j} \in [0, 1]$ represents the strength of the connection between i and j and is based on the culture affinity of the actors. The $s_{i,\ j}$ are calculated by combining the membership functions on different features. For example, if both cultural affinity along feature k and descent were considered critical, one could model cultural affinity through $\chi^k_{i,j}$ and then use an indicator function for whether or not an individual had the desired lineage, such that $s_{i,\ j} = \chi_{i,\ j} \times 1_{descendant}$. In this case, there is no cultural affinity if the individual does not have the proper lineage.

For cultures with multiple features, we use a fuzzy set metric to derive $s_{i,j}$, the relative closeness between two actors. We calculated $s_{i,\ j}$ by examining the feature that maximizes actor i's membership function in actor j's culture.

The relative strength of each relationship $s_{i,\ j}$ is calculated by multiplying values of the actors membership functions in the following manner: take the domain value, d_o, that maximizes actor(i)'s membership function and evaluate actor(j)'s membership function at d_o and then multiply these two values. Defining the strength between nodes in this manner makes the strength network or matrix a directed graph. Formally for one dimension of culture:

$$< e >_{i,j} = f_j(d_o) \times \max[f_i(d_o)]$$

$$\text{for i,j} = 1,\ldots,\text{n nodes}$$

For scenarios where multiple dimensions of culture are to be analyzed the strength operator is calculated for each dimension and then the maximum of the set is chosen, formally:

$$S_{i,j} = \max[<e>^1, <e>^2, \ldots, <e>^n]$$

$$\text{for n} = 1$$

$$S_{i,j} = \max[<e>^1]$$

$$\text{for i,j} = 1, \ldots, \text{n nodes}$$

Alternatively, one could take the average membership across all cultural features as follows:

$$s_{i,j} = \frac{1}{k}\sum_{l=1}^{k}\chi_{i,j}^{l}$$

This flexibility in defining how similarity on different cultural features maps into a single measure of cultural affinity is a major advantage to the approach presented here.

In each period of the model each member of the network decides on an action $a_i \in \{0, 1\}$, where 0 indicates staying home and 1 indicates the person participates in the insurgency in that period. Each person has a threshold $\delta_i \in [0,1]$ and only wants to participate in time t if $\delta_i \leq \Theta_i^t$ where Θ_i^t is the net social influence on agent i. Define Θ_i^t as follows:

$$\Theta_t^t = \frac{1}{n}\sum_{j=1}^{n} z_{i,j}$$

where $z_{i,j}$ is the influence of the j^{th} node on i based on a distance from i to j, $d_{i,j}$, and the strength of the connection between i and j. Define $z_{i,j}$ as follows:

$$z_j = \frac{s_{i,j}}{d_{i,j}}$$

with $d_{i,j}$ being any standard distance metric for a network normalized to [0, 1]. In our model, $d_{i,j}$ is the number of edges between two nodes along the shortest path - the geodesic distance - but other distance metrics could be used. Note that this is a standard threshold model of participation where the key difference is that social influence is now weighted by cultural affinity and social distance.

Simulations

We now examine the dynamics of such a network using a Monte Carlo simulation. We examine the situation with $q = 3$ cultural features defined on a universe of discourse of $m = 8$ integers with continuous membership functions. At the beginning of the simulation the initial connectivity matrix is specified, and participation thresholds are set to 0.1[b]. At each time step the government first selects a member to target at random. Once the member is selected, he is removed with probability p_k. Next, each member of the network looks at all the other agents who participated in the previous period and were not removed, and asks how similar they are to him, so that in $t = 1$ he looks at all members in the initial connectivity matrix. He then weights their influence by similarity and distance. If the weighted sum of these influences is greater than his participation threshold, he participates. If not, he leaves the network. All agents make their decision in this fashion, yielding a new connectivity matrix, and the process starts over again. This continues until there are less than two nodes in the network at which point the network has been disrupted.

To demonstrate the utility and flexibility of the framework, we ran a number of different simulations. Our dependent variable of interest was how long it took the government to disrupt the network. We began with two simple 6-member networks. One was a ring network where all the agents had only two connections, the other was a completely connected network, that is, all the agents were connected to all the other agents.

These simulations used a 1-dimensional cultural space with 8 traits taking values, {0,. . .,7}. For each network, we tested a homogenous culture where individual agents are identical and a heterogeneous culture with cultural traits randomly assigned, resulting in four simulations. For each culture/network we tested both trapezoidal and Gaussian membership functions as presented above with σ set to the standard deviation along the figure.[c] Setting $p_k = 0.3$ we conducted 30 runs of each of the four simulations.

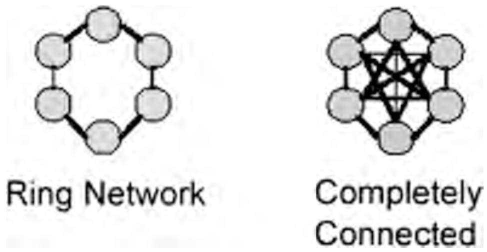

Figure 3.

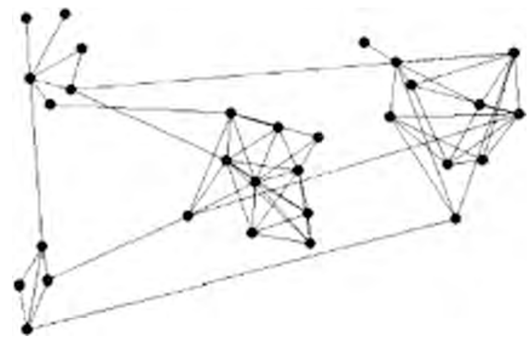

Figure 5.

Under this specification, we found that network topology did not have a significant effect on the mean time to disrupt the network for either distribution of culture. However, under both topologies, the homogenous culture leads to a significantly more robust network. This results is intuitively appealing with respect to culture. Figure (4), shows time to disrupt, as a function of p_k, where the lines represent one standard deviation from the mean.

We conducted a second set of simulations on a 30-member network whose topology is modeled on that of Al Qaeda's core membership as described in Sageman (2004) and Krebs (2002). Figure (5) shows the network we used.

We ran a series of eight experiments to assess how the mean time to disrupt changed with the distribution of culture, membership function, and p_k. We tested all four possible combinations of uniform and normal distributions of culture, and Gaussian and trapezoidal membership functions as defined above, using $S_{i,\ j} = \max[\langle e \rangle^1, \langle e \rangle^2, \langle e \rangle^3]$.[d] Each of the four combinations of culture and membership was run once with p_k equal to post-9/11 efforts against Al Qaeda, $p_k = 0.4$.[e] We ran 300 iterations of each simulation. The results are summarized in table (1).

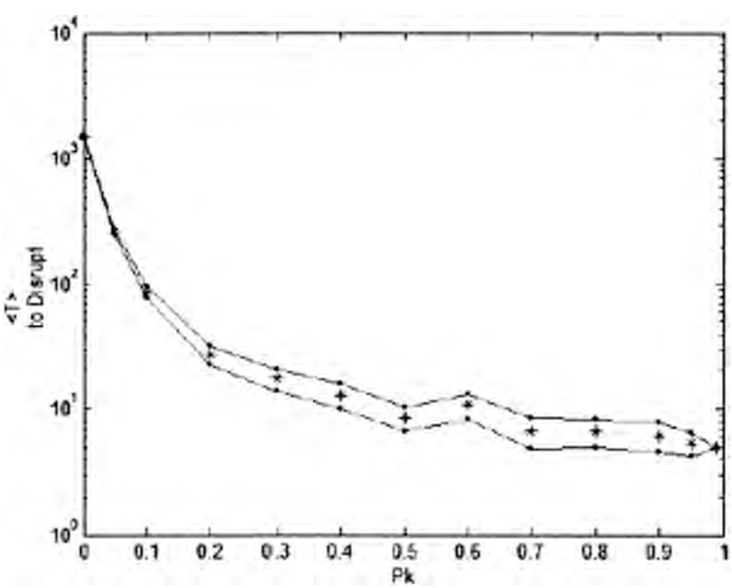

Figure 4.

Table 1. Time to Disrupt

Membership Culture Distribution	$p_k = .4$				$p_k = .05$			
	Trap Norm	Trap Unif	Gauss Norm	Gauss Unif	Trap Norm	Trap Unif	Gauss Norm	Gauss Unif
Mean	38.22 (10.75)	30.63 (8.87)	36.51 (9.97)	31.22 (9.15)	302.31 (120.45)	242.80 (120.44)	293.15 (120.01)	265.15 (108.44)

Note (1): Standard deviations in parenthesis.

We find that on a real world network with realistic parameters and a more coherent culture, the Gaussian membership function leads to a more robust organization. Interestingly, the importance of culture declines as the government's effectiveness increases. Note that the difference in mean time to disrupt between definitions of culture is much smaller at $p_k = .4$ than at $p_k = .05$.

We also see that the definition of cultural similarity matters. With a trapezoidal membership function, the distribution of culture matters less than with a Gaussian membership function. In essence, the trapezoidal membership function is stricter than the Gaussian, influence drops off linearly from the agent's own type to a cut-off point. Our interpretation is that such a rule overwhelms the influence of culture in driving network dynamics. These results demonstrate that this framework can replicate realistic dynamics that cannot be shown with other modeling approaches.

Future work

There are several potential paths that could be explored to extend the proposed modeling approach. One path would explore the dynamics that result when nodes can change. Currently, the fuzzy sets that describe a given agent are static. However, it is likely that as a network comes under attack the nodes evolve, perhaps by strategically manipulating their position in cultural space.[f] This process can be modeled by allowing their underlying descriptive fuzzy sets to change with time or in response to events.

A second path would examine more subtle notions of participation. Our current setup uses a simple threshold model to determine whether a give node participates in the network. If the network influence on the node falls below a given pre-set threshold the agent no longer participates in the network. A potential extension would be to add more complexity into the decision making of the individual nodes using concepts from decision theory or game theory.

Conclusion

We have provided a new formalization for considering the role of culture on social influence processes. This formalization builds on Axelrod's (1997) canonical model of culture as a multi-dimensional space in which actors' types are defined by their position in that space. In contrast to traditional uses of this framework, we suggest that fuzzy sets provide a rigorous and flexible method for defining the role of culture on influence.

This formalization easily admits different definitions of cultural similarity and different distributions of cultural traits. Of particular value is the ability of fuzzy membership functions to deal with dimensions whose metrics are different. Using this method, a model of culture can be developed with some dimensions denoted by scalars, some by ordinal categories, and some by binary distinctions. This flexibility is of great value when developing specific simulations based on expert opinion or combinations of quantitative and qualitative data.

We first modeled a simple network to explore the framework. We then considered the case of an insurgent network where government removes nodes from the network with some probability. We have shown that for a small group, more homogeneous distributions of culture lead to more robust groups, regardless of how the group is connected or how effective government strategy is. We then applied the framework to a network that is topologically similar to Al Qaeda, or at least to Al Qaeda as described in the data used in Sageman (2004) and Krebs (2002). In this setting, we see that cultural homogeneity matters, but how cultural similarity is defined matters more. Specifically, when government slowly removes members, we see that in a heterogeneous culture with its members' traits distributed uniformly throughout the cultural space, more restrictive membership functions lead to more robust networks.

The framework presented here is extremely flexible and easily tailored to meet the specifics of different modeling situations. We hope it will prove useful in future work.

APPENDIX

Below is the pseudo-code for the simulations. Full MATLAB code is available on request from the authors.

1. Generate network
2. Calculate fuzzy sets for culture
3. Calculate distances between nodes
4. Calculate strength matrix
5. Calculate influence
6. For 1 to N runs
7. Loop while number of nodes greater than 2
   (a) Remove node at random
   (b) Update strength and influence data structures
   (c) Remove nodes below threshold
   (d) Update strength and influence data structures
   (e) Test for stopping condition

ENDNOTES

[a] For traits that vary continuously one could use points on a subset of the real line.

[b] where researchers have prior information about the propensity of individuals to participate these participation thresholds can be drawn from the appropriate probability distribution.

[c] Obviously the membership was set to one for the homogeneous culture. For our multi-dimensional simulations σ was calculated for each of the three features.

[d] Unfortunately there is not sufficient open-source data on the cultural attributes that matter for influence within Al Qaeda to use a more specific distribution of cultural traits.

[e] We assessed the empirical p_k using Sageman's data on 366 participants in Al Qaeda and affiliated groups. The data contain dates of entry and exit from the movement as well as method of exit. The empirical p_k is simply the likelihood of being removed by government action.

[f] On the strategic manipulation of identity, see Brass (1979) and Posner (2004).

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